Gaussian Elimination - Exercise Set

Are the following matrices in row-echelon form (REF)?

1) $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

4) $\begin{bmatrix} 3 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

Are the following matrices in reduced row-echelon form (RREF)?

5) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6) $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

7) $\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 1 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

8) $\begin{bmatrix} 1 & 0 & 1 & 2 & 4 \\ 0 & 1 & 0 & 3 & 5 \\ 0 & 0 & 1 & 2 & 2 \end{bmatrix}$

Using elementary row operations, reduce the following matrices to REF.

9) $\begin{bmatrix} 1 & 4 \\ 2 & 12 \end{bmatrix}$

10) $\begin{bmatrix} 1 & 2 & 5 \\ 1 & 3 & 4 \\ 2 & 7 & 13 \end{bmatrix}$

11) $\begin{bmatrix} 3 & 0 & 6 & -6 \\ 0 & 1 & 0 & -3 \\ 2 & -3 & 4 & 5 \end{bmatrix}$

12) $\begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 2 & 0 & 1 \\ 1 & 4 & 1 & 4 \\ -4 & 1 & 3 & 4 \end{bmatrix}$

Using elementary row operations, reduce the following matrices to RREF.

13) $\begin{bmatrix} 3 & 6 & 9 \\ 2 & 4 & 5 \end{bmatrix}$

14) $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 7 & 7 & 4 \end{bmatrix}$

15) $\begin{bmatrix} 2 & 10 & 5 & -1 \\ 0 & 0 & 1 & 4 \\ 1 & 5 & 3 & 2 \end{bmatrix}$

16) $\begin{bmatrix} 1 & 1 & 2 & -2 \\ 0 & 1 & 3 & -4 \\ 1 & 2 & 5 & -6 \\ 2 & 1 & 1 & 0 \end{bmatrix}$

Solve the following systems of equations using Gaussian elimination: (if possible)

17) $\begin{aligned} 2x + 3y &= -6 \\ x - 2y &= 11 \end{aligned}$

18) $\begin{aligned} 2x + 6y &= 24 \\ -x - 3y &= -6 \end{aligned}$

19) $\begin{aligned} 4x + 5y &= -2 \\ -8x - 10y &= 4 \end{aligned}$

20) $\begin{aligned} 3x + 5y &= 0 \\ x + 5y &= -10 \end{aligned}$

21) $\begin{aligned} 5x + 2y &= 4 \\ 3x - y &= 20 \end{aligned}$

22) $\begin{aligned} -x + 3y &= 0 \\ 20x + 10y &= 7 \end{aligned}$

23) $\begin{aligned} x - y &= -4 \\ 10x - 4y &= 2 \end{aligned}$

24) $\begin{aligned} x + 3y &= -15 \\ 3x - y &= 15 \end{aligned}$

25) $\begin{aligned} 4x - 8y &= 10 \\ 12x - 24y &= 20 \end{aligned}$

26) $\begin{aligned} x + 5y &= 8 \\ 4x + 20y &= 32 \end{aligned}$

27) $\begin{aligned} 2x_1 + 6x_2 &= 13 \\ -x_1 + 4x_2 &= 11 \end{aligned}$

28) $\begin{aligned} x_1 - 20x_2 &= 8 \\ 5x_1 - x_2 &= 7 \end{aligned}$

Solve the following systems of equations using Gaussian elimination: (if possible)

29) $\begin{aligned} x_1 + 2x_2 - x_3 &= 6 \\ 2x_1 + x_2 + 4x_3 &= 9 \\ -x_1 - 3x_2 + 5x_3 &= -5 \end{aligned}$

30) $\begin{aligned} x_1 + 2x_2 + 2x_3 &= 11 \\ x_1 + 3x_2 + 13x_3 &= 10 \\ -x_1 + 2x_2 + 12x_3 &= 0 \end{aligned}$

31) $\begin{aligned} x_1 + 4x_2 + 2x_3 &= -3 \\ x_1 + x_2 - x_3 &= 3 \\ -2x_1 - 4x_3 &= 6 \end{aligned}$

32) $\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 3 \\ 2x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1 - x_2 + 2x_3 &= 1 \end{aligned}$

33) $\begin{aligned} x_1 + 4x_3 &= 1 \\ 2x_1 + x_2 + 3x_3 &= 5 \\ 3x_1 + 2x_2 + 2x_3 &= 9 \end{aligned}$

34) $\begin{aligned} x_1 + 2x_2 - 5x_3 &= -1 \\ x_1 + 3x_2 - 7x_3 &= 0 \\ -x_1 + x_2 - 2x_3 &= 3 \end{aligned}$

35) $\begin{aligned} x_1 - x_3 &= 3 \\ -x_1 + 2x_2 - x_3 + 2x_4 &= -6 \\ 2x_1 + 3x_2 + x_3 &= 9 \\ 4x_1 + 4x_3 + 10x_4 &= 15 \end{aligned}$

36) $\begin{aligned} x_1 + x_2 + 3x_3 - x_4 &= 3 \\ -x_1 + 2x_2 - 3x_3 + x_4 &= 0 \\ 5x_1 + 4x_2 + 10x_4 &= -1 \\ 7x_1 + 4x_2 + 6x_3 + 8x_4 &= 0 \end{aligned}$

Solve the following homogeneous systems of equations:

Answers

37) $\begin{aligned} 3x_1 + 2x_2 &= 0 \\ x_1 - 4x_2 &= 0 \end{aligned}$

1) Yes

38) $\begin{aligned} 2x_1 - 5x_2 &= 0 \\ -6x_1 + 15x_2 &= 0 \end{aligned}$

2) Yes

39) $\begin{aligned} x_1 - 3x_2 &= 0 \\ 4x_1 + 7x_2 &= 0 \\ 2x_1 + 8x_2 &= 0 \end{aligned}$

3) No

40) $\begin{aligned} x_1 - 4x_2 - 7x_3 &= 0 \\ -x_1 + 2x_2 + 5x_3 &= 0 \end{aligned}$

4) No

41) $\begin{aligned} x_1 - x_2 - x_3 &= 0 \\ 4x_1 + 2x_2 - 13x_3 &= 0 \\ 2x_1 + 4x_2 - 11x_3 &= 0 \end{aligned}$

5) Yes

42) $\begin{aligned} -x_1 + 2x_2 + 3x_3 &= 0 \\ x_1 + 2x_2 - 6x_3 &= 0 \\ 2x_1 + x_2 + 7x_3 &= 0 \\ x_1 + x_2 + x_3 &= 0 \end{aligned}$

6) Yes

43) $\begin{aligned} 2x_1 + 3x_2 - 7x_3 - 7x_4 &= 0 \\ 3x_1 - 6x_2 + 21x_4 &= 0 \\ -x_1 - 5x_2 + 7x_3 + 14x_4 &= 0 \end{aligned}$

7) Yes

44) $\begin{aligned} x_1 + x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 - 9x_4 &= 0 \\ x_1 + 4x_2 + x_3 - x_4 &= 0 \end{aligned}$

8) No

9) $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

10) $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

11) $\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

12) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

13) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

14) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

15) $\begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

16) $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

17) $x = 3, \quad y = -4.$

18) There's no solution.

19) There are infinitely many solutions. y is arbitrary parameter and
$x = -\dfrac{1}{2} - \dfrac{5}{4}y.$

20) $x = 5, \quad y = -3.$

21) $x = 4, \quad y = -8.$

22) $x = 0.3, \quad y = 0.1.$

23) $x = 3, \quad y = 7.$

24) $x = 3, \quad y = -6.$

25) There's no solution.

26) There are infinitely many solutions. y is arbitrary parameter and $x = 8 - 5y.$

27) $x_1 = -1, \quad x_2 = 2.5.$

28) $x_1 = \dfrac{4}{3}, \quad x_2 = -\dfrac{1}{3}.$

29) $x_1 = 1, \quad x_2 = 3, \quad x_3 = 1.$

30) $x_1 = 3, \quad x_2 = 4.5, \quad x_3 = -0.5.$

31) $x_1 = 1, \quad x_2 = 0, \quad x_3 = -2.$

32) There are infinitely many solutions given by:
$$x_1 = \frac{5}{3} - s - r, \quad x_2 = \frac{2}{3} - s + r, \quad x_3 = r, \quad x_4 = 3s.$$

33) There are infinitely many solutions. z is arbitrary parameter and $x = 1 - 4r, \quad y = 3 + 5r.$

34) $x_1 = -2, \quad x_2 = 3, \quad x_3 = 1.$

35) $x_1 = 4, \quad x_2 = 0, \quad x_3 = 1, \quad x_4 = -0.5.$

36) There's no solution.

37) Trivial solution.

38) $x_1 = 5r, \quad x_2 = 2r.$

39) Trivial solution.

40) $x_1 = 3r, \quad x_2 = -r, \quad x_3 = r.$

41) $x_1 = 5r, \quad x_2 = 3r, \quad x_3 = 2r.$

42) Trivial solution.

43) $x_1 = 2r - s, \quad x_2 = r + 3s, \quad x_3 = r, \quad x_4 = s.$

44) $x_1 = 3r, \quad x_2 = -r, \quad x_3 = 2r, \quad x_4 = r.$