

## Determinant and Matrix Inverse - Exercise Set

---

Find the determinant of the following matrices: (if defined)

1)  $\begin{bmatrix} 3 & 7 \\ -2 & 6 \end{bmatrix}$

2)  $\begin{bmatrix} 10 & 29 \\ 0 & 17 \end{bmatrix}$

3)  $\begin{bmatrix} 5 & 9 & 7 \\ 12 & 0 & -3 \end{bmatrix}$

4)  $\begin{bmatrix} 5 & 1 & 0 \\ 0 & 1 & 4 \\ 3 & 2 & -1 \end{bmatrix}$

5)  $\begin{bmatrix} 8 & 2 & -1 \\ 1 & 0 & 0 \\ 9 & 6 & -5 \end{bmatrix}$

6)  $\begin{bmatrix} -5 & 6 & 3 \\ 0 & 7 & 8 \\ 1 & 2 & 3 \end{bmatrix}$

7)  $\begin{bmatrix} 3 & 1 & -2 & 0 \\ 5 & 1 & 3 & 0 \\ 4 & 2 & 1 & 6 \\ 8 & 2 & 3 & 0 \end{bmatrix}$

8)  $\begin{bmatrix} 7 & 12 & 9 & 20 \\ 0 & 1 & 6 & 8 \\ 0 & 0 & 2 & 16 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Find the inverse of each matrix, if it exists:

9)  $\begin{bmatrix} 1 & 5 \\ 2 & 2 \end{bmatrix}$

10)  $\begin{bmatrix} 13 & 5 \\ 5 & 1 \end{bmatrix}$

11)  $\begin{bmatrix} 4 & 3 \\ 12 & 9 \end{bmatrix}$

12)  $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

13)  $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & -1 \\ 1 & 5 & 2 \end{bmatrix}$

14)  $\begin{bmatrix} 4 & 0 & 3 \\ 1 & 2 & 1 \\ 3 & 0 & 1 \end{bmatrix}$

15)  $\begin{bmatrix} 7 & 2 & 3 \\ -4 & 1 & -5 \\ -1 & 4 & -7 \end{bmatrix}$

16)  $\begin{bmatrix} 11 & 6 & -5 \\ -8 & 2 & 12 \\ 1 & -9 & -13 \end{bmatrix}$

Find the inverse of each matrix, if it exists:

$$17) \begin{bmatrix} 4 & 0 & 0 \\ -2 & 3 & 0 \\ 1 & -9 & 1/2 \end{bmatrix}$$

$$18) \begin{bmatrix} 0.5 & -1 & 5 \\ 1 & 2 & 4 \\ 1 & 6 & 0 \end{bmatrix}$$

$$19) \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 1 & 3 & -2 \\ 1 & -1 & 2 & -3 \\ 0 & 1 & -2 & 6 \end{bmatrix}$$

$$20) \begin{bmatrix} 2 & -2 & 4 & 1 \\ 2 & -1 & 0 & 4 \\ 2 & -3 & 6 & 1 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$

$$21) \begin{bmatrix} 1 & 0 & 2 & 3 \\ 4 & 1 & 3 & -1 \\ 1 & 1 & 2 & 1 \\ -2 & 0 & 1 & 5 \end{bmatrix}$$

$$22) \begin{bmatrix} 0 & 2 & -2 & 0 \\ 0 & 1 & 2 & 4 \\ 2 & 1 & 0 & 4 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

Solve the following systems of equations using inverse of the coefficient matrix:

$$23) \begin{aligned} 3x_1 + 4x_2 - 2x_3 &= 11 \\ -x_1 + 2x_2 + 2x_3 &= 3 \\ x_1 + 2x_2 - x_3 &= 5 \end{aligned}$$

$$24) \begin{aligned} 3x_1 + 4x_2 - 2x_3 &= 16 \\ -x_1 + 2x_2 + 2x_3 &= 18 \\ x_1 + 2x_2 - x_3 &= 7 \end{aligned}$$

$$25) \begin{aligned} 5x_1 + 8x_2 + 5x_3 &= 9 \\ x_1 + 2x_2 + x_3 &= 3 \\ x_2 + x_3 &= 4 \end{aligned}$$

$$26) \begin{aligned} 5x_1 + 8x_2 + 5x_3 &= 2 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + x_3 &= 1 \end{aligned}$$

$$27) \begin{aligned} x_1 + 2x_2 &= 5 \\ 3x_1 + 5x_2 + x_3 &= 15 \\ -x_1 + x_3 &= -2 \end{aligned}$$

$$28) \begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ -x_1 - x_3 &= 0 \\ x_1 + 5x_2 + 2x_3 &= -2 \end{aligned}$$

$$29) \begin{aligned} x_1 + 2x_2 &= 4 \\ 3x_1 + x_2 + 3x_3 - 2x_4 &= -1 \\ x_1 - x_2 + 2x_3 - 3x_4 &= -11 \\ x_2 - 2x_3 + 6x_4 &= 25 \end{aligned}$$

$$30) \begin{aligned} 2x_1 - 2x_2 + 4x_3 + x_4 &= 5 \\ 2x_1 - x_2 + 4x_4 &= 9 \\ 2x_1 - 3x_2 + 5x_3 + x_4 &= 4 \\ x_2 - 2x_4 &= -1 \end{aligned}$$

## Answers

---

1) 32

2) 170

3) Determinant is defined only for square matrices.

4) -33

5) 4

6) 2

7) 24

8) 14

$$9) \frac{1}{8} \begin{bmatrix} -2 & 5 \\ 2 & -1 \end{bmatrix}$$

$$10) \frac{1}{12} \begin{bmatrix} -1 & 5 \\ 5 & -13 \end{bmatrix}$$

11) Inverse does not exist.

$$12) \frac{1}{3} \begin{bmatrix} -5 & 2 & -2 \\ 4 & -1 & 1 \\ -5 & 2 & 1 \end{bmatrix}$$

$$13) \begin{bmatrix} 2.5 & 0.5 & -1 \\ 0.5 & 0.5 & 0 \\ -2.5 & -1.5 & 1 \end{bmatrix}$$

$$14) \frac{1}{10} \begin{bmatrix} -2 & 0 & 6 \\ -2 & 5 & 1 \\ 6 & 0 & -8 \end{bmatrix}$$

15) Inverse does not exist.

16) Inverse does not exist.

$$17) \frac{1}{12} \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ 30 & 72 & 24 \end{bmatrix}$$

$$18) \frac{1}{4} \begin{bmatrix} -24 & 30 & -14 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix}$$

$$19) \frac{1}{7} \begin{bmatrix} 15 & -12 & 28 & 10 \\ -4 & 6 & -14 & -5 \\ -17 & 15 & -28 & -9 \\ -5 & 4 & -7 & -1 \end{bmatrix}$$

$$20) \frac{1}{2} \begin{bmatrix} 6 & -1 & -4 & -1 \\ -12 & 4 & 8 & 6 \\ -7 & 2 & 5 & 3 \\ -6 & 2 & 4 & 2 \end{bmatrix}$$

21) Inverse does not exist.

$$22) \frac{1}{18} \begin{bmatrix} -7 & -7 & 5 & 8 \\ 2 & 2 & -4 & 8 \\ -7 & 2 & -4 & 8 \\ 3 & 3 & 3 & -6 \end{bmatrix}$$

$$23) x_1 = 1, \quad x_2 = 2, \quad x_3 = 0.$$

$$24) x_1 = 2, \quad x_2 = 5, \quad x_3 = 5.$$

$$25) x_1 = -4, \quad x_2 = 3, \quad x_3 = 1.$$

$$26) x_1 = 0, \quad x_2 = -1, \quad x_3 = 2.$$

$$27) x_1 = 3, \quad x_2 = 1, \quad x_3 = 1.$$

$$28) x_1 = 2, \quad x_2 = 0, \quad x_3 = -2.$$

$$29) x_1 = 2, \quad x_2 = 1, \quad x_3 = 0, \quad x_4 = 4.$$

$$30) x_1 = 3, \quad x_2 = 1, \quad x_3 = 0, \quad x_4 = 1.$$