Differentiation Formulas

- Derivative of a constant is zero:

$$\frac{dc}{dx} = 0$$

- Derivative of $f(x) = x$ is 1:

$$\frac{d}{dx} x = 1$$

- Derivative of $f(x) = x^n$ is:

$$\frac{d}{dx} x^n = nx^{n-1}$$

- Derivative of $f(x) = \sqrt{x}$ is:

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

- Derivative of $f(x) = e^x$ is itself:

$$\frac{d}{dx} e^x = e^x$$

- Derivative of $f(x) = e^{ax}$ is:

$$\frac{d}{dx} e^{ax} = ae^{ax}$$

- Derivative of $f(x) = \ln x$ is:

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

- If f is a function and c is a constant, then

$$(cf)' = cf'$$

- If f and g are functions, then

$$(f + g)' = f' + g'$$

Example 1: Evaluate the derivative of $f(x) = x^4$.

Solution: Using the power rule:

$$\begin{aligned}
f' = \frac{df(x)}{dx} &= 4x^{4-1} \\
&= 4x^3
\end{aligned}$$

Example 2: Evaluate the derivative of $f(x) = 3x^5$.

Solution: Using the same rule:

$$\begin{aligned}
f' = \frac{df(x)}{dx} &= 5 \cdot 3x^{5-1} \\
&= 15x^4
\end{aligned}$$

Example 3: Evaluate the derivative of $f(x) = 7x^2 - 4x + 8$.

Solution: We can distribute derivative over addition. Don't forget that the derivative of a constant is zero:

$$\begin{aligned}
f' &= 2 \cdot 7x^{2-1} - 4x^{1-1} + 0 \\
&= 14x - 4
\end{aligned}$$

Example 4: Evaluate the derivative of $f(x) = x\sqrt{x}$.

Solution: The power rule is applicable to this case. Remember that $\sqrt{x} = x^{1/2}$.

$$\begin{aligned}
f' &= \frac{d}{dx} x^{1+1/2} \\
&= \frac{d}{dx} x^{3/2} \\
&= \frac{3}{2} x^{1/2} \\
&= \frac{3}{2} \sqrt{x}
\end{aligned}$$

Example 5: Evaluate the derivative of $f(x) = x^2 - 3\ln x$.

Solution: Once again, use distribution:

$$f' = 2x - \frac{3}{x}$$

Example 6: Evaluate the derivative of $f(x) = 1 + x^3 + e^{-x}$.

Solution: Note that e^{-x} is the same thing as $e^{-1\cdot x}$:

$$f' = 3x^2 - e^{-x}$$

Example 7: Evaluate the derivative of $f(x) = \dfrac{4}{x} - 5e^{7x}$.

Solution: Note that $\dfrac{4}{x}$ is the same thing as $4x^{-1}$:

$$\begin{aligned}
f' &= 4\cdot(-1)\cdot x^{-1-1} - 7\cdot 5e^{7x} \\
&= -\frac{4}{x^2} - 35e^{7x}
\end{aligned}$$

Example 8: Evaluate the derivative of $f(x) = \dfrac{1}{x^2} + \dfrac{1}{e^{3x}}$.

Solution: Note that $\dfrac{1}{e^{3x}}$ is the same thing as e^{-3x}:

$$\begin{aligned}
f' &= \frac{d}{dx}\left(x^{-2} + e^{-3x}\right) \\
&= -\frac{2}{x^3} - 3e^{-3x}
\end{aligned}$$

Derivatives - Exercise Set

Evaluate derivatives of the following functions:

1) $f(x) = 3x^{12}$

2) $f(x) = 2 - 4\sqrt{x}$

3) $f(x) = x^2\sqrt{x}$

4) $f(x) = 7e^x - e^{4x}$

5) $f(x) = x + \dfrac{1}{x}$

6) $f(x) = 3x^4 - 8\ln x$

7) $f(x) = x^3 - e^{-3x}$

8) $f(x) = \dfrac{6}{x^4}$

9) $f(x) = \ln x - \dfrac{x^2}{\sqrt{x}}$

10) $f(x) = \dfrac{e^{-7x}}{7} + \dfrac{e^{5x}}{5} + \dfrac{1}{35}$

Answers

1) $f'(x) = 36x^{11}$

2) $f'(x) = -\dfrac{2}{\sqrt{x}}$

3) $f'(x) = \dfrac{5}{2}x^{3/2}$

4) $f'(x) = 7e^x - 4e^{4x}$

5) $f'(x) = 1 - \dfrac{1}{x^2}$

6) $f'(x) = 12x^3 - \dfrac{8}{x}$

7) $f'(x) = 3x^2 + 3e^{-3x}$

8) $f'(x) = -\dfrac{24}{x^5}$

9) $f'(x) = \dfrac{1}{x} - \dfrac{3}{2}\sqrt{x}$

10) $f'(x) = -e^{-7x} + e^{5x}$