Çankaya University
Department of Mathematics
2017 - 2018 Spring Semester

MATH 108 - Mathematics for Business and Economics - II

First Midterm Examination

1) Consider the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 3 & 4 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} -1 & 3 \\ 5 & 8 \\ 2 & 4 \end{bmatrix}$$

If it is possible, compute the followings. If it is not possible, explain why.

$$2A + B, \qquad AB^T, \qquad (AC)^T, \qquad CA, \qquad BC^T, \qquad CB, \qquad CD + B$$

2) Solve the following system of linear equations by using matrix reduction.

$$\begin{cases} -2x + 4y + z &= 1 \\ x - 3y - z &= 0 \\ x - 2y &= -4 \end{cases}$$

3) Find the solution of the system by using Cramer's Rule.

$$\begin{cases} -x + y + z &= 0 \\ 2x + y + z &= 12 \\ 5x - y - 3z &= 4 \end{cases}$$

4) Find the inverse of the matrix by using Adj(A).

$$A = \begin{bmatrix} 1 & 7 & 3 \\ 5 & 0 & 8 \\ 4 & -1 & 6 \end{bmatrix}.$$

5) Solve the following systems of equations.

a. $\begin{cases} 3x + 2y &= 0 \\ 6x + 4y &= 0 \end{cases}$

b. $\begin{cases} 7x + 8y &= 9 \\ 14x + 16y &= 10 \end{cases}$

c. $\begin{cases} 2x + 3y &= 1 \\ 5x + 10y &= 5 \end{cases}$

Answers

1) $CD = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 5 & 8 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 31 \\ 5 & 8 \end{bmatrix}$

$CD + B = \begin{bmatrix} 15 & 31 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 37 \\ 6 & 10 \end{bmatrix}$

All the other operations are impossible, because matrix dimensions do not agree.

2) Augmented matrix is: $\left[\begin{array}{rrr|r} -2 & 4 & 1 & 1 \\ 1 & -3 & -1 & 0 \\ 1 & -2 & 0 & -4 \end{array}\right]$

Reduction by row operations gives:

$$R_1 \longleftrightarrow R_2 \quad \Longrightarrow \quad \left[\begin{array}{rrr|r} 1 & -3 & -1 & 0 \\ -2 & 4 & 1 & 1 \\ 1 & -2 & 0 & -4 \end{array}\right]$$

$$\begin{array}{l} R_2 :\ +2R_1 \\ R_3 :\ -R_1 \end{array} \quad \Longrightarrow \quad \left[\begin{array}{rrr|r} 1 & -3 & -1 & 0 \\ 0 & -2 & -1 & 1 \\ 0 & 1 & 1 & -4 \end{array}\right]$$

$$R_2 \longleftrightarrow R_3 \quad \Longrightarrow \quad \left[\begin{array}{rrr|r} 1 & -3 & -1 & 0 \\ 0 & 1 & 1 & -4 \\ 0 & -2 & -1 & 1 \end{array}\right]$$

$$\begin{array}{l} R_1 :\ +3R_2 \\ R_3 :\ +2R_2 \end{array} \quad \Longrightarrow \quad \left[\begin{array}{rrr|r} 1 & 0 & 2 & -12 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

$$\begin{array}{l} R_1 :\ -2R_3 \\ R_2 :\ -R_3 \end{array} \quad \Longrightarrow \quad \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

Therefore the solution is: $\begin{array}{rcr} x & = & 2 \\ y & = & 3 \\ z & = & -7 \end{array}$

3) Using Cramer's rule, we obtain:

$$x = \frac{\begin{vmatrix} 0 & 1 & 1 \\ 12 & 1 & 1 \\ 4 & -1 & -3 \end{vmatrix}}{\begin{vmatrix} -1 & 1 & 1 \\ 2 & 1 & 1 \\ 5 & -1 & -3 \end{vmatrix}} = \frac{24}{6} = 4$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 1 \\ 2 & 12 & 1 \\ 5 & 4 & -3 \end{vmatrix}}{\begin{vmatrix} -1 & 1 & 1 \\ 2 & 1 & 1 \\ 5 & -1 & -3 \end{vmatrix}} = \frac{-12}{6} = -2$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & 12 \\ 5 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} -1 & 1 & 1 \\ 2 & 1 & 1 \\ 5 & -1 & -3 \end{vmatrix}} = \frac{36}{6} = 6$$

4) Using the formula for inverse we obtain:

$$A^{-1} = \frac{\mathsf{Adj}(A)}{\mathsf{Det}(A)} = \frac{\begin{bmatrix} 8 & -45 & 56 \\ 2 & -6 & 7 \\ -5 & 29 & -35 \end{bmatrix}}{7} = \begin{bmatrix} \frac{8}{7} & -\frac{45}{7} & 8 \\ \frac{2}{7} & -\frac{6}{7} & 1 \\ -\frac{5}{7} & \frac{29}{7} & -5 \end{bmatrix}$$

5) a. $x = -\frac{2}{3}y$. Infinitely many solutions.

b. $0 = 8$. Impossible. No solution.

c. $x = -1, \quad y = 1$. Unique solution.